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RT-ICAR-NA-2015-2 **Marzo 2015**

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del personale e dei collaboratori dell'ICAR, in alcuni casi in un formato preliminare prima della pubblicazione definitiva in altra sede.

1 Sommario

L' elettrocardiogramma (ECG) è una registrazione dell'attività elettrica dei muscoli del cuore, generalmente utilizzato per l'analisi delle malattie cardiache. Gli impulsi elettrici del cuore possono essere misurati attraverso degli elettrodi, indossabili, durante le attività fisiche o giornaliere. Tuttavia, la qualità di questi segnali acquisiti viene sempre degradata dalla presenza del rumore, durante la fase di acquisizione. In questo lavoro, proponiamo uno schema numerico per il denoising dell' ECG, con un costo computazionale di tipo lineare. La bassa complessità computazionale è dovuto al fatto che il metodo proposto appartiene alla classe dei filtri con risposta impulsiva infinita (IIR). Il principale contributo dello schema proposto è che esso non richiede una diretta applicazione della Fast Fourier Trasform (FFT) per l'eliminazionde delle frequenze del rumore. Inoltre, lo schema proposto, offre la possibilità di una semplice implementazione utile soprattutto per il filtraggio su dispositivi di elaborazione mobile. Esperimenti che testano l'accuratezza e la complessità computazionale sono presentati per testare l'algoritmo.

Abstract

High quality Electrocardiogram (ECG) data is very important because this signal is generally used for the analysis of heart diseases. Wearable sensors are widely adopted for physical activity monitoring and for the provision of healthcare services, but noise always degrades the quality of these signals. In this paper, we propose a novel numerical scheme for ECG Signal denoising with low computational requirements. It is computationally cheap because it belongs to the class of Infinite Impulse Response (IIR) noise reduction algorithms. The main contribution of the proposed scheme is that it does not require a direct application of the Fast Fourier Transform. Moreover, it offers the possibility of implementation on mobile computing devices in an easy way. Experiments on real datasets have been carried out in order to test the algorithm.

2 Introduction

The accurate analysis of noisy Electrocardiogram (ECG) data is a very interesting challenge. This is especially true in relation to the pervasive use of wearable healthcare monitoring systems [1], where physiological data acquired from real life can be used for remote healthcare scenarios, for the early analysis of diseases, as in e.g. [2], or for the highlighting of correlations between health and a correct lifestyle, as in e.g. [3]. The ECG biomedical signal is composed of weak non-stationary data which are affected by various types of noises: power line interference, baseline drift, electromyography interference and sensor contact noise. Generally, a good denoising scheme has the capability of removing noises, from the acquired signal, by filtering the data and by ensuring a result as close as possible to the unknown original signal. In literature, there are numerous research papers devoted to this problem, including: adaptive filtering [4, 5], Wiener filtering [6], Empirical Mode Decomposition [7], and wavelet denoising [8] (for other methods see [9]). In ECG filtering, a crucial problem is the preservation of the sharp, that is achieved by several algorithms, for example by non local means filtering [10]. Unfortunately, these schemes are quite computationally expensive. In this paper, starting from a methodology based on Recursive Filtering, applied successfully by the authors in another research field [11, 12, 13], we propose a novel numerical scheme for ECG Signal denoising with low computational costs, in terms of memory and time. Our approach is based on the analysis of the signal in the Fourier domain, but it does not require a direct application of the (Fast) Fourier Transform. With respect to other methods, we compute the solution with only a few floating point operations. This feature makes the scheme suitable for direct implementation in applications on mobile devices, dedicated to the real time filtering of biomedical signals. In order to test the algorithm, we report the performance metrics achieved by applying the proposed methodology to some records of the database PhysioNet [14], that offers a large collection of recorded physiological ECG signals. The paper is organized as follows: in section 2 we give some preliminary mathematical considerations; section 3 is devoted to the numerical scheme; in section 4 we report the numerical experiments and, finally, in section 5 we draw our conclusions.

3 Mathematical Preliminaries

In this section we present a scheme for the filtering of digital signals with a computational cost of $O(n)$ floating point operations. This scheme is based on an approximation of a continuos convolution with a suitable homogeneous and isotropic correlation function h (e.g. [15]) . Let s_0 denote a real function such that

$$s_0 = s + \epsilon$$

with s the original signal and ϵ a noise function. In computer vision, researchers as in (e.g. [16]) use the convolution of s_0 with a function h, Lebesgue integrable, to obtain a denoised function s_h by s_0, i.e.

$$s_h(t) = [h * s_0](t) = \int_{-\infty}^{+\infty} h(t-x)s_0(x)dx, \quad \forall t \in \mathbb{R}. \tag{1}$$

The focus of this section is to determine suitable properties for the function h in order to determine a denoising scheme. The scheme has to eliminate noises ϵ s.t. the frequency spectrum is wandering in a limited range. To achieve this aim we use the following mathematical tools:

- the Fourier Transform $\mathcal{F}$ of a signal f

$$F(\omega) = \mathcal{F}(f)(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} f(x)e^{-i\omega x}dx, \quad \forall \omega \in \mathbb{R}; \tag{2}$$

- the Fourier anti-Transform $\mathcal{F}^{-1}$ of F

$$f(t) = \mathcal{F}^{-1}(F)(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} F(x)e^{itx}dx, \quad \forall t \ \in \mathbb{R}; \tag{3}$$

- the convolution theorem

$$\mathcal{F}(h * f) = \mathcal{F}(h) \cdot \mathcal{F}(F) = H \cdot F; \tag{4}$$

The main idea of this work is to find a suitable convolution kernel h, to denoise s_0, starting from its Fourier Transform $H = \mathcal{F}(h)$ in the Fourier domain. Now, let us suppose that

$$S_h = \mathcal{F}(s_h), \quad S_0 = \mathcal{F}(s_0), \quad S = \mathcal{F}(s) \quad \text{and} \quad \mathcal{E} = \mathcal{F}(\epsilon).$$

If we assume that the Fourier Transform $H = \mathcal{F}(h)$ of h is such that:

$$\begin{cases} H \cdot S = S, \\ H \cdot \mathcal{E} = 0, \end{cases} \tag{5}$$

then, it can be easily proved that it holds that:

$$s_h = h * s_0 \equiv s. \tag{6}$$

Observing that, if the original and noise signals, s and ϵ, have the property

$$supp\, S \cap supp\, \mathcal{E} = \varnothing, \tag{7}$$

then, it is possible to obtain an infinite class of functions that satisfy condition (5). Then, in these hypotheses, the original signal s can be completely restored by the convolution in (6). Nevertheless, as shown in the next example, a satisfying solution s_h (an approximation of s) can be achieved without assuming (7), and by filtering the signal s_0 by means of a certain convolution kernel.

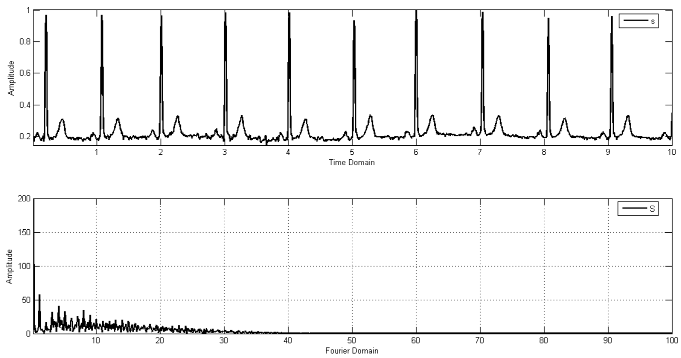

Figure 1: Top: an ECG signal s. Bottom: $\mathcal{F}-$Transform $S = \mathcal{F}(s)$ of s

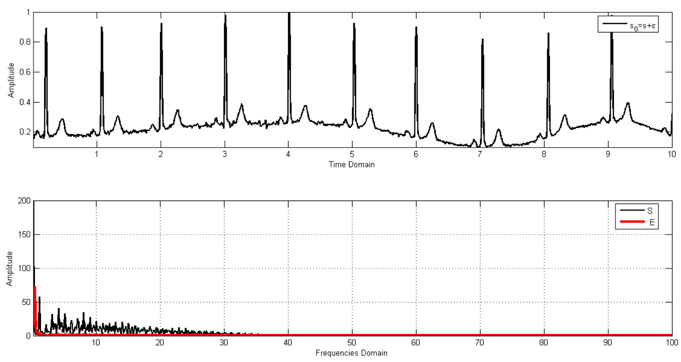

Figure 2: Top: the noisy ECG signal $s_0 = s + \epsilon$. Bottom: $\mathcal{F}-$Transform $S_0 = \mathcal{F}(s_0)$ of s_0 , sum of $S = \mathcal{F}(s)$ (black line) and of $\mathcal{E} = \mathcal{F}(\epsilon)$ (red line)

Now, let s be an original ECG signal (top of Figure 1) and $s_0 = s+\epsilon$ (top of Figure 2) the noisy signal, obtained by the noise function ϵ. Here we take ϵ as the Base Line Wander noise (e.g.[7]) Assuming that the interval $[\mu-\sigma,\ \mu+\sigma]$ contains the support of the Fourier Transform $\mathcal{E}$, i.e. $supp\, \mathcal{E} \subseteq [\mu-\sigma,\ \mu+\sigma]$, then we consider a convolution kernel $h = \mathcal{F}^{-1}(H)$, where H is set as

$$H(\omega) = \begin{cases} 0 & \text{if } \omega \in [\mu - \sigma,\ \mu + \sigma] \\ 1 & otherwise \end{cases} \tag{8}$$

With these assumptions, a way of obtaining a denoised signal s_h is given by the following steps:

(i) $\mathcal{F}$-Transform s_0, to obtain $S_0 = \mathcal{F}(s_0) = S + \mathcal{E}$ (S and S_0 are shown, respectively, in the bottom of Figures 1 and 2);

(ii) define the function H as in (8);

(iii) multiply H for S_0, to achieve $S_h = H \cdot S_0$ (top of Figure 3);

(iv) $\mathcal{F}^{-1}$-Transform $S_h = H \cdot S_0$, to extract from s_0 the signal $s_h = \mathcal{F}^{-1}(S_h)$ (bottom of Figure 3).

All these figures have been generated by a `Matlab` code (Code 1) that uses the Fast Fourier Transform (FFT) and Inverse Fast Fourier Transform (IFFT) (e.g. Van Loan), in order to implement steps (i)–(iv). This code takes into account the symmetry of $supp\, \mathcal{E}$ in the Fourier domain, when ϵ is a periodic function. In particular in Code 1, the parameters μ, σ of (8) are set as $\mu = 0,4\ Hz$ and $\sigma = 0,3\ Hz$.

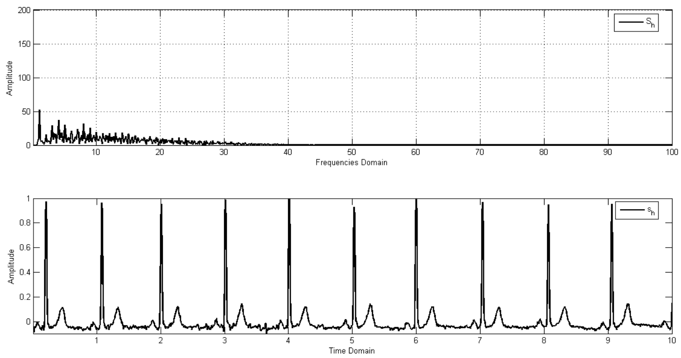

Figure 3: Top: the restored signal s_h. Bottom: $\mathcal{F}-$Transform $S_h = \mathcal{F}(s_h)$

```
function s_h = FFT_Filter(s_0,mu,sigma);
% Inputs: % s_0 vector noised input data
fl=(mu-sigma)/fs %normalized lower frequency and;
fu=(mu+sigma)/fs %normalized upper frequency to cut;
S_0 = fft(s_0,n,2);
H=ones(1,n);
k =floor(fl*n):floor(fu*n)
H(1,k) = 0;   H(1,n-k+2) = 0;
S_h(1,:)=H(1,:).*S_0(1,:);
s_h= real(ifft(S_h,n,2));
```

Code 1: Matlab Code of a FFT Filter for Base Line Wander in ECG signals

We highlight that the bottom of Figure 2 indicates that $supp\, S \cap supp\, \mathcal{E} \neq \varnothing$, but after applying the Code 1, in which H is set as 0 also on $supp\, S \cap supp\, \mathcal{E}$, the signal s_h can anyway be considered an accurate approximation of s. However, the mathematical form of H in (8), and its discontinuities, prevent us from determining our numerical scheme. Therefore, for our purpose, as we will show in the next section, instead of H, we will use a function $\tilde{H}$ that emulates the properties of H. This function is defined as

$$\tilde{H}(\omega) = \frac{(\omega - \mu)^2}{2\sigma^2 + (\omega - \mu)^2}, \quad \forall \omega \in \mathbb{R}. \tag{9}$$

and is a rational approximation of the function

$$G(\omega) = 1 - e^{(-(\omega-\mu)^2/2\sigma^2)}, \quad \forall \omega \in \mathbb{R}. \tag{10}$$

Notice that $\tilde{H}$ is obtained from G, by taking the first two terms in the Taylor expansion of the following exponential function

$$e^{((\omega-\mu)^2/2\sigma^2)} = \sum_{i=0}^{+\infty} \frac{1}{i!} \left(\frac{(\omega - \mu)^2}{2\sigma^2} \right)^i \tag{11}$$

The functions $\tilde{H}(\omega)$ and $G(\omega)$ share the following properties:

1. $0 \leq G(\omega), \tilde{H}(\omega) < 1, \quad \forall\, \omega \in \mathbb{R}$;
2. $\tilde{H}(\mu) = G(\mu) = 0$;
3. $\lim_{\omega \to \pm\infty} \tilde{H}(\omega) = \lim_{\omega \to \pm\infty} G(\omega) = 1$;
4. $\tilde{H}(\omega)$ and $G(\omega)$ are symmetrical with respect to the axis $\omega = \mu$;
5. $\tilde{H}(\mu \pm \sigma) = 1/3$;
6. $\tilde{H}(\omega) = \tilde{H}_l(\omega) \cdot \tilde{H}_r(\omega), \quad \forall \omega \in \mathbb{R}$, where we set

$$\tilde{H}_l(\omega) = \frac{-i\omega + i\mu}{-i\omega + (\sqrt{2}\sigma + i\mu)}, \qquad \tilde{H}_r(\omega) = \frac{i\omega - i\mu}{i\omega + (\sqrt{2}\sigma - i\mu)}. \tag{12}$$

It is still possible to compute the denoised signal $s_{\tilde{h}}$ by means of Code 1, replacing $\tilde{H}$ in (9) instead of H in (8).
The choice of function $\tilde{H}$ in (9) can determine the noisy ϵ, even if its frequency spectrum is wandering in the interval $[\mu - \sigma, \mu + \sigma]$. Moreover, with this function, it is possible to provide a numerical scheme to obtain the denoised function $s_{\tilde{h}}$ from s_0 as shown in next section.

4 A novel $\mathcal{O}(n)$ Numerical Scheme

In this section, we introduce the derivation of our scheme for the denoising of digital signals. This algorithm is based on the Infinite Impulse Response (IIR) Gaussian Recursive Filter of [17] and [18]. It reduces the effects of additive noise functions ϵ on the original signal s, when $supp\, \mathcal{E} \subset [\mu - \sigma, \mu + \sigma]$. In terms of floating point operations, this algorithm is faster than FFT.

As we will show later, it has a computational cost of $\mathcal{O}(n)$, while FFT has a cost of $n\log(n)$.
As a preliminary remark, we observe that if $S_0 = \mathcal{F}(s_0)$ and if $\tilde{h} = \mathcal{F}^{-1}(\tilde{H})$, with $\tilde{H}$ defined as in (9), then for the function $s_{\tilde{h}} = \tilde{h} * s_0$ it holds that

$$s_{\tilde{h}} = \tilde{h}_l * (\tilde{h}_r * s_0) \tag{13}$$

where functions $\tilde{h}_l$ and $\tilde{h}_r$ are defined as

$$\tilde{h}_l = \mathcal{F}^{-1}(\tilde{H}_l) \quad \text{and} \quad \tilde{h}_r = \mathcal{F}^{-1}(\tilde{H}_r),$$

and functions $\tilde{H}_l$ and $\tilde{H}_r$ are as in (12). In order to determine a numerical scheme, we need to sample the signals s, s_0. From now on, we will consider the discrete signals

$$\vec{s}_0 = (s_0[1], \ldots, s_0[n]), \quad \vec{s} = (s[1], \ldots, s[n]) \quad \text{and} \quad \vec{\epsilon}_0 = (\epsilon[1], \ldots, \epsilon[n])$$

obtained from s_0, s, ϵ, by using an uniform discretization with stepsize τ, i.e.

$$s_0[j] = s_0(j\tau), \quad s[j] = s(j\tau), \quad \epsilon[j] = \epsilon(j\tau), \qquad j = 1, ..., n. \tag{14}$$

It is well known that for the frequency range of discrete signals it holds

$$-\frac{\pi}{\tau} \leq \omega \leq \frac{\pi}{\tau} \quad \Longleftrightarrow \quad -\frac{\phi_d}{2} \leq \phi \leq \frac{\phi_d}{2} \tag{15}$$

where $\phi_d = 1/\tau$ and $\omega = 2\pi\phi$.
Our numerical scheme is based on the discretization of the continuous scheme,

$$\mathcal{S}_{\tilde{h}} = \tilde{H}_l \cdot \tilde{H}_r \cdot S_0. \tag{16}$$

where $\tilde{H}_l$ and $\tilde{H}_r$ in (12) represent respectively cause and anti-cause stable differential equations for continuous signals that can be transformed into causal and anti-causal difference equations for discrete signals by means of standard techniques. The classic methods are bilinear transform, finite differences, the zero-pole matching method and others (e.g. [19]), For our scheme we have used the zero-pole matching method. This approach has the advantage of transforming stable differential equations into stable difference equations and of not using approximations like the others. Given a polynomial

$$p(\omega) = i\omega + (\alpha \pm i\beta), \quad \omega \in \mathbb{C}; \tag{17}$$

the zero-pole matching method exploits the following position:

$$z = e^{i\omega\tau}, \tag{18}$$

Using equation (18), the zeros of $p(\omega)$ are transformed into points belonging to the unit circle $C = \{z \in \mathbb{C} \ : \|z\|_2 = 1\}$ of the complex plane that are used to build the polynomial in z variable:

$$p(z) = 1 - 2e^{-2\alpha\tau}cos(\beta\tau)z^{-1} + e^{-2\alpha\tau}z^{-2}. \tag{19}$$

Let $\vec{s}_0$ be a discrete signal with sampling step τ, and applying the zero-pole matching method to continuous scheme $S_{\tilde{h}} = \tilde{H}_l \cdot \tilde{H}_r \cdot S_0$, we obtain the following forward and backward numerical denoising scheme.

Denoising Numerical Scheme

$$\boxed{\begin{array}{l} p_{\tilde{h}}[j] = b_0 s_0[j] + b_1 s_0[j-1] + b_2 s_0[j-2] + \\ \qquad\qquad a_1 p_{\tilde{h}}[j-1] + a_2 p_{\tilde{h}}[j-2] \quad j = 3, ..., n \ : +1 \\ s_{\tilde{h}}[j] = b_0 p_{\tilde{h}}[j] + b_1 p_{\tilde{h}}[j+1] + b_2 p_{\tilde{h}}[j+2] + \\ \qquad\qquad a_1 s_{\tilde{h}}[j+1] + a_2 s_{\tilde{h}}[j+2] \quad j = n-2, ..., 1 \ : -1 \end{array}} \tag{20}$$

where the recursive scheme coefficients in (20) are:

$$b_0 = 1, \quad b_1 = -2\cos(\mu\tau), \quad b_2 = 1 \quad and \quad a_1 = 2e^{-\sqrt{2}\sigma\tau}\cos(\mu\tau) \quad a_2 = -e^{-2.\sqrt{2}\sigma\tau}. \tag{21}$$

The computational cost of the forward and backward difference scheme in (20) is :

$$\mathcal{T}(n) = 18 \ n \ \ t_{calc}. \tag{22}$$

where n is the size of the discrete signals $\vec{s}_0, \vec{p}_{\tilde{h}}$ and $\vec{s}_{\tilde{h}}$ and t_{calc} is the time for a floating point operation.
To close the equations in (20) at the borders, we have to fix the initial conditions. We have supposed the signals $\vec{s}_0$, $\vec{p}_{\tilde{h}}$ and $\vec{s}_{\tilde{h}}$ to exist and assume a constant value also for $j < 1$ and $j > n$. The border conditions for $\vec{s}_0$ are the following: $s_0[j] = s_0[1]$ for all $j < 1$, then for $\vec{p}_{\tilde{h}}$, it holds that $p_{\tilde{h}}[j] = (b_0 + b_1 + b_2)s_0[1]/(1 - a_1 - a_2)$ $\forall j < 1$, i.e. the steady-state response to an infinite stream of $s_0[1]$ value using the first equation in (20). Similarly, $\forall j > n$, the $p_{\tilde{h}}[j]$ assumes a constant value $p_{\tilde{h}}[n]$, then $s_{\tilde{h}}[j] = (b_0 + b_1 + b_2)p_{\tilde{h}}[n]/(1 - a_1 - a_2)$ for all $j > n$, i.e. the steady-state response to an infinite stream of $p[n]$ value using the second equation in (20). Hence to complete the statements in (20) we have fixed the following heuristic initial conditions for the forward and backward procedures:

$$\boxed{\begin{array}{ll} forward\ conditions & backward\ conditions \\ p_{\tilde{h}}[0] = \dfrac{(b_0 + b_1 + b_2)}{(1 - a_1 - a_2)} s_0[1] & s_{\tilde{h}}[n+1] = \dfrac{(b_0 + b_1 + b_2)}{(1 - a_1 - a_2)} p_{\tilde{h}}[n] \\ p_{\tilde{h}}[-1] = \dfrac{(b_0 + b_1 + b_2)}{(1 - a_1 - a_2)} s_0[1] & s_{\tilde{h}}[n+2] = \dfrac{(b_0 + b_1 + b_2)}{(1 - a_1 - a_2)} p_{\tilde{h}}[n]. \end{array}} \tag{23}$$

We conclude the section by giving a possible scheme of the algorithm that uses the forward and backward equations in (20). The algorithm can be used for the denoising of a discrete signal $\vec{s}_0 = \vec{s} + \vec{\epsilon}$, s.t., it is known the frequency spectrum range of $\vec{\epsilon}$. The scheme of the proposed denoising algorithm, that from now we indicate as the Recursive Filter (RF), is the following:

Algorithm 1 Scheme of the Recursive Filter (RF)

`Input:` $\vec{s}_0$ μ, σ `Output:` $\vec{s}_{\tilde{h}}$

`Compute` b_0, b_1, b_2, a_1 and a_2 `by means of the formulas` (21).
$p_{\tilde{h}}[-1] = ((b_0 + b_1 + b_2)/(1 - a_1 - a_2))s_0[1]$.
$p_{\tilde{h}}[0] = ((b_0 + b_1 + b_2)/(1 - a_1 - a_2))s_0[1]$.
for j=1,2...,n
$p_{\tilde{h}}[j] = b_0 s_0[j] + b_1 s_0[j-1] + b_2 s_0[j-2] + a_1 p_{\tilde{h}}[j-1] + a_2 p_{\tilde{h}}[j-2]$
endfor
$s_{\tilde{h}}[n+2] = ((b_0 + b_1 + b_2)/(1 - a_1 - a_2))p_{\tilde{h}}[n]$.
$s_{\tilde{h}}[n+1] = ((b_0 + b_1 + b_2)/(1 - a_1 - a_2))p_{\tilde{h}}[n]$.
for j=n,n-1...,1
$s_{\tilde{h}}[j] = b_0 p_{\tilde{h}}[j] + b_1 p_{\tilde{h}}[j+1] + b_2 p_{\tilde{h}}[j+2] + a_1 s_{\tilde{h}}[j+1] + a_2 s_{\tilde{h}}[j+2]$
endfor
Return $\vec{s}_{\tilde{h}}$

5 Numerical Experiments on the ECGs

In this section, we compared the results, on accuracy and computational cost measures, of RF to them of a method that exploits the FFT as in Code 1. a first order zero-phase lowpass filter (LPF) (e.g. [19]) and a single stage of median or moving average filtering (BPF) (e.g. [20, 21]). In our experiments, we have used data from the Physionet Long-Term ST Database. The Long-Term ST Database contains 86 lengthy ECG recordings of 80 human subjects, chosen to exhibit a variety of events of ST segment changes, including ischemic ST episodes, axis-related non-ischemic ST episodes, episodes of slow ST level drift, and episodes containing mixtures of these phenomena. The database was created to support development and evaluation of algorithms capable accurately differentiating of ischemic and non-ischemic ST events, as well as basic research into mechanisms and dynamics of myocardial ischemia. Then a pre-processing phase is due to eliminate the additional artifact on ECGs, before using these algorithms, for efficient distinction between physiological and pathological events .

The ECG signals used, from the Long-Term ST database, last 3600 seconds (s) and are: s20011, s20051, s20061, s20071 and s20081 and s20121. These recordings were sampled at 250 Hz using 11-bit A/D converters. We have processed both the actual Physionet recordings (converted to milli volts) and the signals with synthetic Base Line Wander noise added. Base Line Wander is caused by respiration or patient movement which create problems in the detection of peaks. Due to wander the T peak would be higher than the R peak and might be detected as an R peak instead. The amplitude variation is 15% of the peak to peak ECG amplitude. It is normally considered below 1 Hz. The FFT method, LPF and BPF algorithms above represent in literature the fastest and most accurate approaches for the the denoising of an ECG with a Base Line Wander noise.

Let $\vec{s}_0 = \vec{s} + \vec{\epsilon}$ with $\vec{s}$, $\vec{s}_0$ and $\vec{\epsilon}$ described in Section 3 and shown respectively in the top and center of Figure 4. In the bottom of the same picture, we have reported $\vec{s}_{\tilde{h}}$, the RF application to $\vec{s}_0$. The first impression is that the RF reconstructs quite successfully the signal $\vec{s}$ also in a part of the ECG where there is a pathological event (from 1,5 s to 2,5 s).

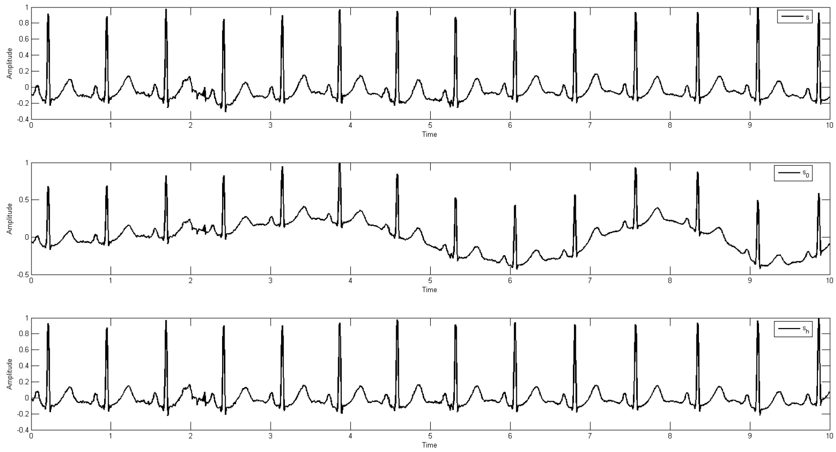

Figure 4: The top of the figure shows $\vec{s}$ that contains the first 10 seconds of s20011. The center of the figure represents the noisy signal $\vec{s}_0$. The bottom of the figure represents the denoised signal $\vec{s}_{\tilde{h}}$

As in [22], we quantify the denoising performance in terms of the Signal-to-Noise ratio (SNR) (in decibels):

$$SNR = 10 \log_{10} \left(\frac{\sum_{i=1}^{n} \left(s_0[i] - s[i] \right)^2}{\sum_{i=1}^{n} \left(s_f[i] - s[i] \right)^2} \right) \tag{24}$$

In (24), $\vec{s}_f$ is one of the denoised signals obtained by means of the filter above and n is the length of $\vec{s}$, $\vec{s}_0$ and $\vec{s}_f$. We highlight that here we have assumed that the Physionet signals $\vec{s}$ are the true signals; in reality these signals also contain noise, which the metric above neglects.
In Table (5), we have reported the SNR measures, varying the ECGs chosen and varying the filters considered. First of all, we can observe from Table

Table 1: Signal-to-Noise ratio (SNR) (in decibels)

	SNR	$s20011$	$s20051$	$s20061$	$s20071$	$s20081$	$s20121$
	FFT	$15,73$	$14,15$	$17,72$	$14,44$	$13,85$	$17,65$
:	LPF	$13,96$	$12,75$	$15,71$	$13,64$	$13,21$	$14,27$
	BLF	$10,37$	$11,18$	$10,90$	$9,54$	$10,79$	$11,35$
	RF	$14,39$	$13,33$	$15,88$	$13,69$	$13,17$	$14,41$

1, that the most accurate filter is, in any case, the FFT method, with in second position the RF.

In the left of Figure 5, we show the average results in terms of execution time and memory usage of the examined filters, for the denoising of ECG s20011 with size n=900000. The experiments have been carried out using an Asus CPU Intel(R) Core(TM) i7-4510U CPU 2.00 GHZ -2.60 GHZ, RAM 6 GB. Figure 5 shows that RF and LPF have the lowest time while the RF and the FFT method exploit the lowest amount of memory.
Taking into consideration the SNR measure in Table 5 and the computational cost tests in the left of Figure 5 then RF has the possibility of implementation on mobile computing devices for the denoising of ECG signals. Finally, we report in the right of Figure 5 A screenshot of an Android application for the de-noising of an ECG based on RF. It proves to be very fast on many devices of the latest generation, also for long ECGs recordings. At the moment it does not work in real time but we are researching the optimal boundary conditions for a variable time window for that purpose.

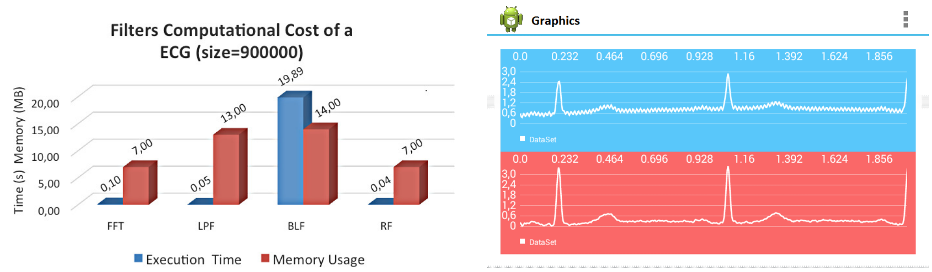

Figure 5: Left: The figure shows the computational cost (execution time and memory usage) of FFT, LPF, BPF and RF to denoise an ECG (s20011) with a size of 900000 samples. Right: a screenshot of an Android application for the de-noising of an ECG based on RF

6 Conclusions

In this paper we have described the development and implementation of a scheme (RF) for ECG Signal Denoising.
Numerical experiments on some ECGs from the Physionet Long-Term ST Database have demonstrated that our RF can significantly reduce the total computational cost of denoising compared to other efficient filters, while maintaining the same level of accuracy.
In addition we provide the theoretical development of the new scheme, based on the study by [17], who formulated the RF in the context of signal processing to eliminate high frequency noise. We have adapted this for ECG signal denoising, providing a description of the process to obtain the RF coefficients for different kinds of noise known.
The RF is faster because it has only a computational cost of $\mathcal{O}(n)$. Therefore, we can implement it on mobile computing devices to achieve improvements in e-health care.

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